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Designing a Hybrid Model of Data Envelopment Analysis with Taguchi Approach to Optimize Multiple Response Banks Performance

Mohammad Reza Jafari*[a], Gholamreza Esmailian[b], Meysam Jafari Eskandari[c], Reza Tavakkoli-Moghaddam[d]

[a] *PhD student, Department of Industrial Engineering, Payame Noor University, Tehran, Iran.*
[b] *Assistant Professor, Department of Industrial Engineering, Payame Noor University, Tehran, Iran.*
[c] *Assistant Professor, Department of Industrial Engineering, Payame Noor University, Tehran, Iran.*
[d] *Professor, Department of Industrial Engineering, University of Tehran, Tehran, Iran.*

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Abstract

Many organizations (including banks) have a multi-step process and their operations are a continuous process in successive periods. The Taguchi method is an efficient way to optimize a single quality response. However, in practice most products / processes have more than one qualitative response. Recently, the multi-answer question in the Taguchi method has attracted considerable research attention. Therefore, this study presents an efficient approach to multi-response problem solving in Taguchi method using hybrid data envelopment analysis (DEA) model. Each experiment is discussed in the Taguchi Orthogonal Array (OA) as a decision unit (DMU) with multiple input and / or output response sets. Each DMU is evaluated by a hybrid model. The sequential DUM productivity value is then used to decide the optimal factor levels for the multi-response problem. The computational results showed that the proposed approach provides the most anticipated improvement in PCA, DEA (DEAR) and other available techniques. The suggestion may be of great help to managers in solving multi-response problems in production programs in the Taguchi method.

Keywords: Multilevel Programming, Taguchi Method, Data Envelopment Analysis, Hybrid Model, Bank.
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*Mohammad Reza Jafari
Email address: jafari_mohammadreza@yahoo.com, gre@pnu.ac.ir, jafari@bmn.ir, tavakoli@ut.ac.ir (Mohammad Reza Jafari*[a], Gholamreza Esmailian[b], Meysam Jafari Eskandari[c], Reza Tavakkoli-Moghaddam[d])

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1. Introduction

Banking industry is one of the most complicated industries in the world which is held responsible for the assets and the wealth of a country. Banks play a key role in economic growth of a country, due to the variety in financial and credential services, both in macroeconomics and microeconomics. Banks and financial institutions collect resources by increasing deposits and make them available for investors in terms of facilities. The assets of banks is the main resource for product purchase and services and granted loans are the main resource of credits for all economic units such as families, jobs, firms and government. Nowadays, banks present a wide range of products and services, from simple account opening, retirement programs to mutual investing, home loans, consumer loans and other activities. Although internet based banking services and deal channels has dramatically increased, clients in person still hold a considerable percentage of added value in banking services. In the recent years, increasing threats of globalization and growth of non-bank financial institutions compelled banks to establish research centers to analyze the performance and take proportionate action in order to survive in the market in the rivalry with other banks.

One of the main issues in banking industry, especially banks with a lot of branches, is the lack of a logical performance evaluation method. One of the traditional techniques is one level performance evaluation method which is unable to provide accurate managerial reports on advantages and disadvantages of competitive strategies. The technique is also unable to accurately detect inefficiencies of units of a bank. On the other hand, multi-level data envelopment analysis is able to overcome the issues. Therefore, performance is one of the main factors affecting the growth evaluation and facilitation in businesses. In the market of Iran, businesses and firms should make proper strategies in order to compete on selling product and providing services by setting the minimum price. Increasing the efficiency of a unit and optimum usage of resources has attracted a lot of attention, especially after sanctions are imposed against Iran and the concept of resistance economy started. Performance evaluation of different units of an organization by a strategic pattern, should be systematic within a logical framework. One of the well-known methods in performance evaluation, is the data envelopment analysis. Although, the method is well-known, the method is unable to prioritize inputs and is also unable to measure the effect of each input on the performance of units. In fact, data envelopment analysis is only able to discriminate efficient units from inefficient units.

Lack of selection of the main condition in process factors is a costly mistake in a competitive market. The general purpose of a stable design is to find settings for control factors so as to make response sensitive to changes of the noise variables while preserving an acceptable mean level of response.

Taguchi method [1] is a widely used method for stable design which uses an Orthogonal Array (OA) to obtain reliable information about design parameter with minimum time and resource. The method also employs signal-to-noise (S/N) ratio for interpretation of empirical data and optimization of performance. However, the method is only employed to optimize controllable factor levels which is a single response issue. Intense competitive in today's market compelled industries to produce products with more than one response. To resolve the multi-response issue, Taguchi method leverages a surrogate relation between quality decrease and efficiency, but the approach proposed by Phadke et al. [2] may produce contradictory optimal factor level to resolve the multi-response issue and increases the uncertainty in decision making process. Shiau et al. [3] defined a weight for each response and then used the sum of weighted responses. Tong et al. [4] used S/N ratio in normal weighted total quality loss. However, weight determination is difficult for each quality response in real-world scenarios. Reddy et al. [5] leveraged regression-based methods for optimization of multi-response problem. Unfortunately, regression approaches increases the complexity of computations and requires statistical skills. Furthermore, Antony et al [6] employed Principal Component Analysis (PCA) to convert multi-response together in some unrelated cases. Then, principal components are used for searching optimal factor level of multiple responses. PCA considers an assumption in which random variables are usually multiple variables limited by error expressions in real-world scenarios. Yu et al. [7] presented a strategy called Taguchi fuzzy-neural network using genetic algorithm. Jeyapaul et al. [8] developed a genetic algorithm and Al-rafaie et al. [9] leveraged gray analysis for optimization of multiple response issue in Taguchi method. In fact, controlled computation techniques of genetic algorithm, neural networks and gray analysis is not fully understood by many managers. Data Envelopment Analysis (DEA) presented by Charnes et al. [10] is a fractional mathematical programming technique for evaluating relative efficiency of Decision Making Units (DMUs) with various inputs and outputs. DEA combines various inputs and outputs into a performance measure called relative efficiency for DMU. DEA techniques can be divided into two categories. First category of methods includes comparative information which is presented by a decision maker or a specialist, while the second category does not include comparative information, such as the method proposed by Angulo-Meza et al. [11]. First category includes direct weight limitations (Dyson et al. [12]), Conical ratio model (charnes et la. [13]) and value efficiency analysis (halme et al. [14]). However, there are issues related to mentality in the methods. First, comparative information could be wrong or biased. Second, lack of consensus between experts or decision makers has a negative effect on study, while techniques of second category which does not depend on previous information, have effective results and hence, improves the decision making process. One of the techniques of second category is hybrid model in mutual evaluation. Unlike traditional techniques of DEA, hybrid model makes discrimination between effective DMUs and instead of self-evaluation, makes peer evaluation available for DMU. Due to the advantages and the decrease in uncertainty in decision making process, this study employs hybrid model to resolve the multiple response issue in Taguchi method. Each experiment of Taguchi array as a DMU is discussed with a set of multiple response as input/output. Hybrid model is then used for performance evaluation of each DMU and also for decision making in optimal factor level of multiple response. The rest of the paper is organized in 5 sections. DEA is first introduced in data envelopment analysis section.

2. Data Envelopment Analysis

Data envelopment analysis is broadly used for performance evaluation of a set of DMUs with numerous inputs and outputs in organizational level, such as banks, hospitals and universities (Charnes et al. [15]). Widely used DEA technique is actually an extension of CCR model presented by Charnes et al. [10].

2.1. CCR Model

CCR Model along with the comparison with another set of DMUs with the same set of input and output measures the relative efficiency of each DMU. DMU_o represents a DMU under evaluation. If DMU_o has a relative efficiency equal to 1, it is considered efficient. Otherwise, it is considered inefficient. However, CCR model is unable to discriminate between the efficient DMUs, because the relative efficiency grades may be equal for multiple DMUs. On the other hand, Khouja et al. [16] leveraged two stage approach to select high technology robot production from a list of possible technologies. In the first stage, the efficiency of robot is detected by CCR model and then is evaluated by multi-feature decision making model in the second stage. Liao [17] employed neural networks to predict response for all combinations of factor level. Then, CCR model is used for decision making on settings of optimal factor. However, Baker and Talluri [18] evaluated the robot selection issue in Khoja study and showed that CCR model has an essence problem in which DMU detection reports false efficiency grades with unreal weighting schema. Furthermore, it may lead to multiple optimal solutions. To cover the deficiency of the model, this study employs a hybrid model in mutual evaluation to measure and compare the performance of a set of DMUs.

2.2. Hybrid Model in Mutual Evaluation

Mutual evaluation technique is first introduced by sexton et al. [19] which uses DEA in peer evaluation, instead of computed self-evaluation by CCR model. Self-evaluation is used for measuring efficiency of DMU_o using its input and output weights, while peer evaluation means that DMU_o is evaluated according to optimal weight schema of other DMUs. The mean of the obtained efficiencies is mutual evaluation. However, multiple optimal solutions exist which changes the mutual evaluation. The issue is resolved by the introduction of a secondary target function using the hybrid model. The main purpose of hybrid model is obtaining a weight schema of DMU_o which is optimal in CCR model but the secondary purpose is maximization of cross efficiencies of DMUs which is proposed by Angulo-Meza and Lins [11]. The technique is shown by two models I and II. By any of the models, once optimal values *uro* and *vio* or $\mathrm{u}^*{}_{ro}$ and v^*io are are obtained, the mutual evaluation of DMU_o is calculated. In this paper, E_{oj} represents the mutual efficiency of DMUj and is computed according to the optimal weights of DMU_o. E_{oj} is computed as follows:

$$E_{oj} = \sum_{r=1}^{s} u^*_{ro} y_{rj} / \sum_{i=1}^{m} v^*_{io} x_{ij} \quad j \neq 0 \tag{2.1}$$

keepaspectratio=false]image2Once E_{oj} values are computed, a matrix called "mutual efficiency matrix" is created. Here, e_j as the mean of mutual efficiencies of DMUj is computed as follows:

$$\mathrm{e}_j = \sum_{o \neq j} E_{oj}/(n-1) \quad j = 1, \ldots, n \tag{2.2}$$

e_j values are used for the performance comparison of nDMU. Unlike CCR model, hybrid model increases the discrimination of efficient DMUs by accepting values more than one. The hybrid models are used to resolve the multiple response issue in Taguchi method which is described in the following section.

3. The Proposed Method

The products have quality responses which describes their performance according to the needs or customer expectations. In general, a qualitative product/process feature (QCH) or response is divided into three main types: Smaller-The-Better (STB) response, Nominal-The-Best (NTB) responses and Larger-The-Better (LTB) responses. STB responses have ideal target of zero, NTB feature has a specific defined target by user and finally, LTB responses have an infinite ideal target or state. In practice, multiple responses of product/process does not necessarily belongs to the type of response. In this study, it is assumed that responses are not related to each other. The stages of the proposed method is presented to resolve the multi-response issue by Taguchi method in the following:
Stage 1 suppose experiments are conducted using OA Taguchi. Each experiment is treated as a DMU, the relative efficiency is defined as the sum of weighted outputs divided to number of weighted inputs. As usual, higher efficiency shows the better performance which is obtained as sum of weighted outputs increases or the sum of weighted inputs decreases. To increase the relative efficiency of a DMU and to achieve the target of each quality response, input and output of each DMU is tuned as below:

1. If all responses are of STB type, then all responses are defined as input and a unit (one) is defined as output. On the contrary, of all responses are of LTB type, then all responses are defined as output, while a unit (one) is tuned as input for all DMUs. In other words, efficiency is improved by decreasing denominator in the former case, and increasing numerator in the latter case.
2. If all responses are of NTB type, then quality loss Lj of DMUj is computed as below (Tong et al. [20]):

$$L_j = c\left(s_j^2/\bar{y}_j^2\right) \quad j = 1, \ldots, n \tag{3.1}$$

Which c is the quality loss, while y_j and s_j are the mean and the standard deviation of responses of DMUj. Since the aim is to minimize the quality loss, L_j values are defined as input and the output of all DMUs are set to 1.
3. If responses are a combination of all three types, values of STB type response and L_j values of NTB type response are set as input, while LTB type response(s) are set as output for all DMUs.

Stage 2 The relative efficiency E_o of each DMU is evaluated by resolving the input-oriented CCR model.
Stage 3 The optimal weight of input and output $u^*{}_{ro}$ and $v^*{}_{io}$ are estimated by resolving I model of each DMU_o. Then, the grades of each DMU is computed using the equation (2.1) of mutual evaluation E_{oj}. The mean of mutual efficiency e_j is computed using equation (2.2).
Stage 4 The sequential value of e_j is defined to optimize the performance and to avoid the big values of e_j in selection of optimal levels. The sequential value is the actually the grading of e_j values by which the sequential value of 1 is assigned to the minimum value of e_j, while the sequential value of n is assigned to the maximum value of e_j. AOV_{fl} is the mean of sequential values of f factor in level *1*. The value of AOV_{fl} is computed for each factor level. The higher AOV_{fl} is usually proportionate to the better performance. Therefore, the optimal factor level l^* is the level which maximizes the value of AOV_{fl}.

$$l^* = \left\{ l | \max_{l} \{AOV_{fl}\} \right\} \quad \forall f \tag{3.2}$$

If an equality relationship occurs for an agent in optimal level selection, then the optimal level for the factor is a level with maximum expected progress.
Stage 5 repeat the stages 3 and 4 for performance evaluation of each DMU using model II instead of model I.
Stage 6 predicted improvement for each response is estimated in order to determine optimal factor level and then the predicted progress by the proposed method is compared with the predicted progress by other approaches for each response. If responses have different quality loss, then quality loss is computed for each response. Otherwise, S/N ratio is computed for each response using the equations below:

For STB type responses:

$$\mathrm{S/N}ratio = -10 \log_{10} \left(\frac{1}{K} \sum_{k=1}^{K} y_k^2 \right) \tag{3.3}$$

For NTB type responses:

$$\mathrm{S/N}ratio = 10 \log_{10} \frac{\mu^2}{\sigma^2} \tag{3.4}$$

For LTB type responses:

$$S/N ratio = -10\log_{10}\left(\frac{1}{K}\sum_{k=1}^{K}\frac{1}{y_k^2}\right) \tag{3.5}$$

in which L is the response iteration. μ and σ are the mean response and the standard deviation. The mean S/N ratio is obtained by each factor level. In the following section, different applications of the proposed method is described.

4. Optimization of Efficient Loans

In the following case studies, qualitative coefficients for numerous responses are equal. Therefore, S/N ratio is used to estimate the predicted improvement for each response.

Table 1: Empricial data for efficient loans

Response	Level	Factor A	B	C	D	E	F	Optimal Factor Level using Taguchi	Total Mean
Facilities	I	**35.12**	31.61	34.39	31.68	30.52	27.04	A1B3C1D2E2F3	31.52
	2	34.91	30.70	27.86	**34.70**	**32.87**	33.67		
	3	24.52	32.24	32.30	28.16	31.16	**33.85**		
Probable loss	I	**-24.23**	**-27.55**	**-39.03**	**-39.20**	-51.53	-45.56	A1B1C1D1E2F2	-45.36
	2	-80.II	-47.44	-55.99	-46.85	**-40.54**	**-41.58**		
	3	-61.76	-61.10	-41.07	-50.04	-44.03	-48.95		
Profit	I	28.76	32.03	32.80	32.21	34.06	33.81	A3B3C2D3E3F3	34.12
	2	34.13	34.78	**35.29**	34.53	33.99	34.10		
	3	**39.46**	**35.54**	34.25	35.61	34.30	**34.44**		

Phadke et al. [2] employed Taguchi method to improve the quality of efficient loans for three responses. Probable loss (STB), facilities (NTB) and profit (LTB) are the main responses.
Six process factors are evaluated simultaneously including: (A) work force, (B) economic growth rate, (C) equipment, (D) selected inputs, (E) inflation rate and (F) branch area using an array of L18 ($2^1 \times 3^7$) which is shown in Table 1. Taguchi method employs S/N ratio for decision making on optimal factors of each response. In this method, higher S/N ratio shows better performance. The mean S/N ratio of each factor level for each response type is computed using the appropriate formula from equation (3.3) to equation (3.5). As shown in Table 2, optimal factor level for facilities, probable loss and cost is A1B3C1D2E2F3, A1B1C1D1E2F2, A3B3C2D3E3F3, respectively. It is a challenging task to select one of the three combinations of optimal factor level for simultaneous optimization of three responses. The simultaneous optimization of the three responses by the proposed method is described in the following stages:

Table 2: The mean S/N ratio for efficient loans

DMU1	Control Factor"								Input		Output	Standard Efficiency ($£_0$)
	e	A	B	C	D	E	F	*e*	Facilities ($x1j$)	Probable Loss ($x2j$)	Profit ($y1j$)	
DMU1	I	1	1	1	1	1	1	1	0.00030	0.67	14.5	1.00000
DMU2	1	1	2	2	2	2	2	2	0.00027	36.22	36.6	0.38025
DMU3	1	1	3	3	3	3	3	3	0.00025	135.78	41.4	0.22037
DMU4	1	2	1	1	2	2	3	3	0.00006	17.00	36.1	1.00000
DMU5	1	2	2	2	3	3	1	1	0.00719	1,087.78	73.0	0.02626
DMU6	1	2	3	3	1	1	2	2	0.00051	839.89	49.5	0.09788
DMU7	1	3	1	2	1	3	2	3	0.00726	776.33	76.6	0.03359
DMU8	1	3	2	3	2	1	3	1	0.00520	2,065.33	105.4	0.03032
DMU9	1	3	3	1	3	2	1	2	0.00087	2,200	115.0	0.13343
DMU10	2	1	1	3	3	2	2	1	0.00206	0.89	24.8	1.00000
DMU11	2	1	2	1	l	3	3	2	0.00013	1.00	20.0	1.00000
DMU12	2	1	3	2	2	1	1	3	0.00016	246.56	39.0	0.25200
DMU13	2	2	1	2	3	1	3	2	0.00062	150.11	53.l	0.16001
DMU14	2	2	2	3	1	2	1	3	0.00005	44.44	45.7	1.00000
DMU15	2	2	3	1	2	3	2	1	0.00018	1,359.44	54.8	0.30722
DMU16	2	3	1	3	2	3	1	2	0.00065	14.33	76.8	0.67157
DMU17	2	3	2	1	3	1	2	3	0.00629	2,201.22	105.3	0.02609
DMU18	2	3	3	2	1	2	3	I	0.01438	3,333.33	91.4	0.01227

Table 3: The obtained optimal weight using the hybrid model for efficient loans

Response	Level	Factor A	B	C	D	E	F	Optimal Factor Level using Taguchi / Total Mean
Facilities	I	35.12	31.61	34.39	31.68	30.52	27.04	
	2	34.91	30.70	27.86	**34.70**	**32.87**	33.67	
	3	24.52	32.24	32.30	28.16	31.16	**33.85**	
Probable loss	I	**-24.23**	**-27.55**	**-39.03**	**-39.20**	-51.53	-45.56	
	2	-SO.II	-47.44	-55.99	-46.85	**-40.54**	**-41.58**	
	3	-61.76	-61.10	-41.07	-50.04	-44.03	-48.95	
Profit		28.76	32.03	32.80	32.21	34.06	33.81	
	2	34.13	34.78	**35.29**	34.53	33.99	34.10	
	3	**39.46**	**35.54**	34.25	**35.61**	**34.30**	**34.44**	

4.1. Mutual Efficiency Matrix by Hybrid Model I and II for Efficient Loans

4.1.1. Predicted progress for efficient loans

Stage 1 an array L18 ($2^1 \times 3^7$) includes 18 experiments. As shown in column 1 of Table 1, each experiment is considered as a DMU. Facilities are computed by equation (3.2) and the inputs are

defined by the probable loss. However, the profit is defined as output for all DMUs.
Stage 2 the standard efficiency $Eo(o = 1, \ldots, 18)$ is computed by resolving CCR model for each DMU and is shown in column 1. Note that all values of Eo is between 0 and 1, while the value of Eo for each DMU1, DMU4, DMU10, DMU11 and DMU14 is equal to 1. Therefore, the DMUs are efficient as CCR. The deficiency of CCR model is effective in discrimination between DMUs.
Stage 3 model I is chosen to evaluate u^*_{1j} and v^*_{2j} and u^*_{1j} values for each DMUj. The results are shown in columns with "model I" title in table 3. The values v^*_{21} and v^*_{11} and u^*_{11} for DMU1 is obtained 0.0, 21.6778700, 0.0004485, respectively by solving model I as stated in the following.

4.1.2. Model I

The model is stated as below:

$$\begin{aligned}
&\sum_{r=1}^{s}\left(u_{ro}\cdot\sum_{j\neq o}y_{rj}\right)-\sum_{i=1}^{m}\left(v_{io}\cdot\sum_{j\neq o}x_{ij}\right)\\
&\sum_{i=1}^{m}\left(v_{io}\cdot\sum_{j\neq o}x_{ij}\right)=1\\
&\sum_{r=1}^{s}u_{ro}y_{rj}-\sum_{i=1}^{m}v_{io}x_{ij}\leq 0,\forall j\neq o\\
&\sum_{r=1}^{s}u_{ro}y_{ro}-E_o\cdot\sum_{i=1}^{m}v_{io}x_{io}=0\\
&u_{ro},v_{io}\geq 0,\quad \forall r,\forall i
\end{aligned}\tag{4.1}$$

In this model, the decision making variables are *uro* and *vio.* The purpose of target function is probably to maximize the mutual efficiency of DMUs while measuring by their best weight. First limitation ensures that total weighted inputs and outputs of other (n-1) DMUs using input and output weights of DMUo equals to 1 for converting target function to linear function.
Second limitation guarantees that the each score of relative efficiency for all DMUs except DMU$_o$ is less than 1. Third limitation ensures that the relative efficiency DMU$_o$ computed by CCR model equals to E$_o$. The final limitation ensures that all optimal values for decision making variables *uro* and *vio* is positive.

4.1.3. Model II

In model II, the target function is maximization of sum of weighted outputs, while the limitations is the same as model I. The formulas are mathematically expressed below:

$$\begin{aligned}
&\sum_{r=1}^{s}\left(u_{ro}\cdot\sum_{j\neq o}y_{rj}\right)\\
&\sum_{i=1}^{m}\left(v_{io}\cdot\sum_{j\neq o}x_{ij}\right)=1\\
&\sum_{r=1}^{s}u_{ro}y_{rj}-\sum_{i=1}^{m}v_{io}x_{ij}\leq 0,\quad \forall j\neq o\\
&\sum_{r=1}^{s}u_{ro}y_{ro}-E_o\cdot\sum_{i=1}^{m}v_{io}x_{io}=0\\
&u_{ro},v_{io}\geq 0
\end{aligned} \tag{4.2}$$

As in model I, decision making variable are *uro* and *vio*.

$$\begin{aligned}
&\text{Max}\quad u_{11}\cdot\sum_{j=2}^{18}y_{1j}\\
&-\left(v_{11}\cdot\sum_{j=2}^{18}x_{1j}+v_{21}\cdot\sum_{j=2}^{18}x_{2j}\right)\\
&\text{subject to }\sum_{i=1}^{2}\left(v_{i1}\cdot\sum_{j=2}^{18}x_{ij}\right)=1\\
&u_{11}y_{1j}-\sum_{i=1}^{2}v_{i1}x_{ij}\leq\delta\\
&j=2,\dots,18\\
&u_{11}y_{11}-\textstyle\sum_{i=1}^{2}v_{i1}x_{i1}=0\\
&u_{11},v_{11},v_{21}\geq 0
\end{aligned} \tag{4.3}$$

To prevent the unusable solution, the right hand side of the second limitation is equal or less than δ scale and as shown in column 2 of Table 3, it is close to 0. The values of $v^*{}_{1j}$, $v^*{}_{2j}$ and $\mathrm{u}^*{}_{1j}$ are computed for other DMU17. The values of E_{oj} and e_j are computed for each DMU 18. Table 4 shows the mutual efficiency matrix. The mutual efficiency E2,1 of DMU1 is computed as follows using the optimal weight schema DMU2 0.1356. In Table 1, DMU1 inputs are computed as 0.00030 and 0.67, respectively, while the output is estimated as 14.5. In Table 3, using the pattern I of hybrid model, values $v^*{}_{12}$, $v^*{}_{22}$ and $\mathrm{u}^*{}_{12}$ are computed as 21.6637800, 0.0 and 0.0000608, respectively. By substituting the values in equation (2.1) following expression is obtained:

$$E_{2,1}=(0.0000608\times 14.5)/(21.6637800\times 0.0003+0.0\times 0.67)=0.1356 \tag{4.4}$$

In a similar approach, values of E_{o1} DMU1 are evaluated using optimal weights of DMU3 to DMU18. The mean mutual efficiency of DMU1 e_1 using the equation (2.2) is as follows:

$$\mathrm{e}_1=\sum_{o=2}^{18}E_{o1}/(18-1)=21.0133 \tag{4.5}$$

The values of e_j for DMU2 to DMU18 is computed as the same. Unlike CCR model, the values of E_{oj} for some DMUs are more than 1, according to Table 4. For example, the values of $E3{,}1$ and $E6{,}1$ are 15.641 and 35.9422, respectively. Furthermore, a DMU known as efficient CCR by CCR model has unequal ej values and it is not as efficient as hybrid model and it represents the efficiency of hybrid model in discrimination between efficient DMUs.

Stage 4 The sequential values for all values of e_j is mentioned in the last row of Table 4. The minimum value of e_j has a sequential value of 1, while maximum value of e_j has a sequential value of 18. The values of AOV_{f1} is computed for all factor levels using the sequential values and it is shown in Fig. 1. For example, AOV_{A1} as the efficiency level 1 of factor A is computed as the mean of sequential values for DMU1, DMU2, DMU3, DMU10, DMU11 and DMU12 and then is divided to 6. AOV$A1$ (=13.5) is numerically obtained by 6/ (17 + 10 + 9 + 18 + 16 + 11). The values of AOV_{fl} is also obtained for other similar factors. In Fig 1, the factor level which maximizes the efficiency level is considered as the optimal level for the factor. Therefore, A1B1C1D2E2F2 is the combination of factor levels which optimizes the three responses, simultaneously.

Stage 5 The values of $v^*{}_{1j}$, $v^*{}_{2j}$ and $\mathrm{u}^*{}_{1j}$ is computed by resolving the model II for each DMU and are mentioned in columns named "model II" in Table 3. For example, the values of $v^*{}_{21}$, $v^*{}_{11}$ and $\mathrm{u}^*{}_{11}$ for DMU1 are obtained by resolving the model below.

$$\mathrm{Max}\, u_{11} \cdot \sum_{j=2}^{18} y_{1j} \tag{4.6}$$

which is established under the expression below:

$$\begin{aligned} &\sum_{i=1}^{2}\left(v_{i1}\cdot\sum_{j=2}^{18} x_{ij}\right) = 1 \\ &u_{11}y_{1j} - \sum_{i=1}^{2} v_{i1}x_{ij} \le \delta \\ &j = 2,\ldots,18 \\ &u_{11}y_{11} - \sum_{i=1}^{2} v_{i1}x_{i1} = 0 \\ &u_{11}, v_{11}, v_{21} \ge 0 \end{aligned} \tag{4.7}$$

which shows that both of models provide the same values of $v^*{}_{1j}$, $v^*{}_{2j}$ and $\mathrm{u}^*{}_{1j}$ for all DMUs. The mutual efficiency matrix corresponds to the similar model II. Therefore, A1B1C1 D2E2F2 is a combination of factor levels for simultaneous optimization of three responses by model II. By this stage, it would be obvious which of the models I or II is appropriate for resolving multi-response issue in Taguchi method. In the stage 6, the effectiveness of the proposed method on optimization of efficient loans is investigated. The mean S/N ratio in any factor level for all factor levels is computed and it is shown in Table 2. The other approaches in the literature including normal sum of weight loss [4], PCA [20] and DEAR [21] are shown. Maximum predicted improvements in facilities and the probable loss conforms to the proposed method. However, maximum predicted improvement in profit conforms to sum of weighted normal quality loss. Nevertheless, the proposed method provides the total maximum predicted progress among all the techniques. As a result, the superior proposed method, sum of normal weighted loss, PCA and DEAR are efficient in resolving multiple response issue in Taguchi method for efficient loans.

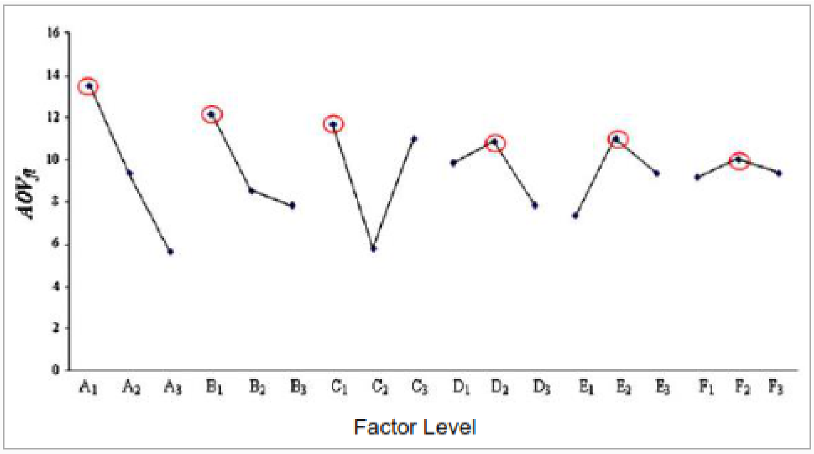

Figure 1: The obtained optimal factor level for efficient loans

5. Conclusion

The purpose of performance evaluation is detecting units with weak performance in order to help managers on improving the units. The main issue of inefficient or low-efficient units is detecting the main source and reasons for deficiency in performance. Banking industry has a special position among different industries and hence, the efficiency of banks has been in a priority for bank managers, economic officials and also people as bank clients. On the other hand, the weak efficiency of a bank leads to reduction in competence power and finally reduction in market share which is not actually pleasant for bank managers. This study presents an approach to resolve the multiple response issue in Taguchi method using a hybrid model in DEA. Advantages of the proposed method are:

- Efficiency: The proposed method is highly effective on multiple response issue in Taguchi method, because maximum predicted improvement is provided for the three cases.

- Prior information: Unlike Taguchi method and S/N weight ratio, the proposed method does not require any prior information about weight or priority.

- Discrimination: The proposed method is not flexible considering the assumptions, while PCA considers the assumptions.

- Simplicity: Unlike GA, neural network, gray analysis and regression method, the proposed method is easily understood and administered by bank managers.

It is required to select appropriate input and output variables of a bank according to the policies and goals of the bank in order to correctly and realistically evaluate the efficiency. The general approach of banks must be defined clearly considering its general strategy in order to evaluate the efficiency. In data envelopment analysis of a study, if a single number changes among all the data, or if a single unit of data changes among all the variables, bank efficiency would be affected. One of the effective factors on accurate efficiency computation is the appropriate selection of inputs and outputs. A solution to the issue is to use the existing information in balance sheet and also the profit and loss in bank bills which includes the inputs and outputs of bank. Therefore, it is recommended to gather all the information about balance sheet and also the profit and loss of banks in order to compute the efficiency. The efficiency of branches is evaluated by data envelopment analysis of non-radial models and is compared with radial models. The efficiency of branches is evaluated by an intermediate

approach and is compared with production approach. Each index can be divided into low cost and high cost deposit considering the appropriate weight according to the cost they make for the bank. The weighted mean of the index is computed using pair-wise comparison. Investigated variables of the study are of quantitative type, however in future studies qualitative variables such as employee satisfaction or quality of service can be used for efficiency evaluation of bank management.
In future studies, more diverse indexes can be used for performance evaluation of banks. It is recommended to evaluate the customer satisfaction by total resources to the number of branches, total resources to the number of employees, total facilities to the number of branches and total facilities to the number of employees. It is recommended to evaluate the supervisor satisfaction by the index of special facilities (including interest-free loan, house loan, elite loan, production loan) with preferred interest rates and it is recommended to evaluate the employee satisfaction by the index of job satisfaction (quit index, absence and etc.).

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